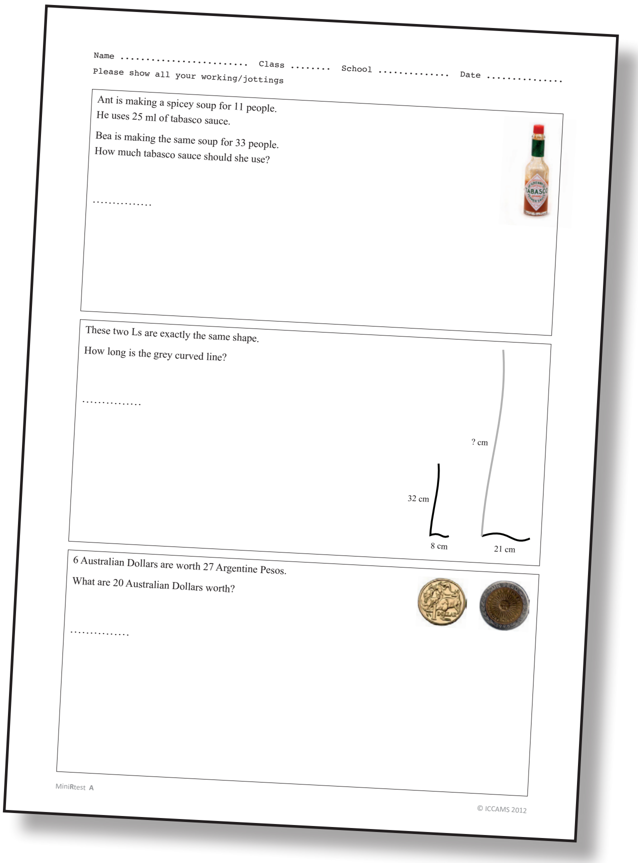
Mini Ratio Test (Versions A and B)

Ask students to work through version A or B of this mini test, before starting on Lessons 2A and 2B. Distribute Versions A and B randomly to students in the class.

This can be done during another lesson, or for homework. **It should about 10 minutes**.


Name .... Class .... School .... Date ....
Please show all your working/jottings

Ant is making a spicey soup for 11 people.
He uses 25 ml of tabasco sauce.

Bea is making the same soup for 33 people.
How much tabasco sauce should she use?

....

These two Ls are exactly the same shape.

How long is the grey curved line?

....

? cm

32 cm

8 cm

21 cm

6 Australian Dollars are worth 27 Argentine Pesos.

What are 20 Australian Dollars worth?

....

MiniRtest A

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Commentary

The aim of this mini test is to get a sense of the range of methods that students use to solve ratio tasks, and to see whether choice of method (and task difficulty) is influenced by context and the numbers in the task.

The test involves three very different contexts - a soup recipe, a geometric enlargement, and currency conversion.

The first two tasks can be solved by using a simple multiplier (of ×3 and ×4 respectively), whereas the third task involves the more complex multipliers ×3⅓ or ×4½. The scalar and functional relationships between the numbers are shown on the ratio tables on the next page.

Go through the students' responses. Make a brief record of the methods used and whether they were successful. Look out for these methods (as applied to the Soup task on Version A):

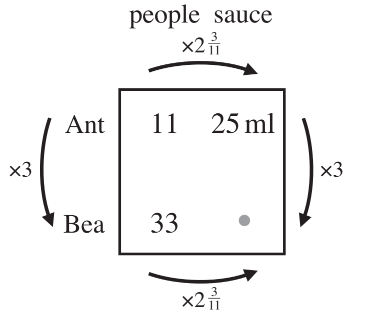
- scaling (3 times as many people will need 3 times as much sauce)
- rated addition (11 people need 25 ml, so 22 need 50, so 33 need 75)
- the unitary method (11 people need 25 ml, so 1 person needs $2\frac{3}{11}$ ml, so 33 people need 75 ml)
- constructing and solving an equation, eg 11/33 =25/x
- the rule of 3 (desired amount = 33 × 25 ÷ 11)
- an inappropriate addition strategy (22 extra people need 22 extra ml, making 47 ml in all).

Mini Ratio Test
Version A

Soup:

scalar relation: $\times 3$

functional rel: $\times 2\frac{3}{11}$



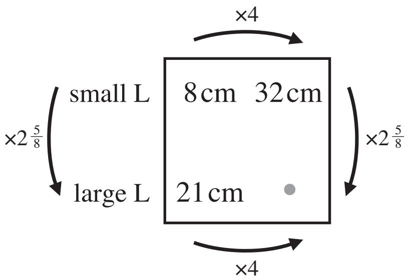
Ant is making a spicy soup for 11 people.
He uses 25 ml of tabasco sauce.

Bea is making the same soup for 33 people.
How much tabasco sauce should she use?

Curved L:

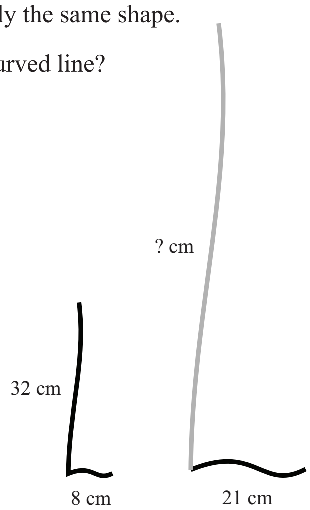
scalar relation: $\times 2\frac{5}{8}$

functional rel: $\times 4$



These two Ls are exactly the same shape.

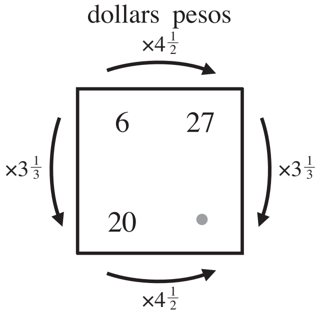
How long is the grey curved line?



Dollars:

scalar relation: $\times 3\frac{1}{3}$

functional rel: $\times 4\frac{1}{2}$



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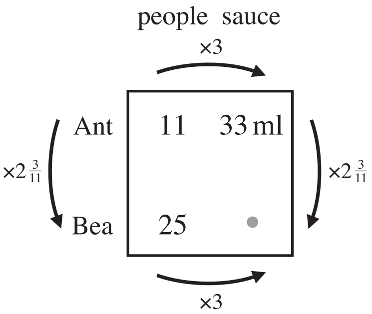
Mini Ratio Test
Version B

Soup:

scalar relation: $\times 2\frac{3}{11}$

functional rel: $\times 3$



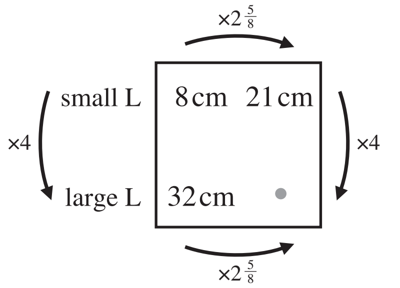
Ant is making a spicy soup for 11 people.
He uses 33 ml of tabasco sauce.

Bea is making the same soup for 25 people.
How much tabasco sauce should she use?

Curved L:

scalar relation: $\times 4$

functional rel: $\times 2\frac{5}{8}$



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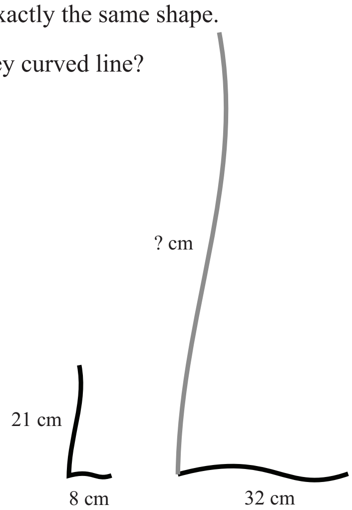
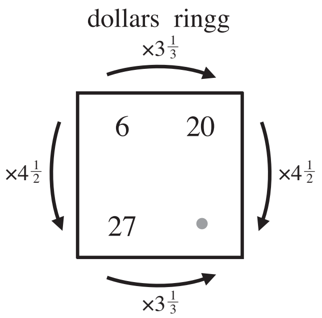


Dollars:

scalar relation: $\times 4\frac{1}{2}$

functional rel: $\times 3\frac{1}{3}$



6 Australian Dollars are worth 20 Malaysian Ringgits.

What are 27 Australian Dollars worth?

Background

Mini Ratio Test data

We distributed items from the Mini Ratio Test, together with other such items, randomly to students within 29 classes, mostly Year 8, across a total of 14 schools. Each item was given to over 70 students.

The table below, left, shows the percentage of students (in these parallel samples of about 75 students) who answered the item correctly (or with just minor computation errors).

The table below, right, shows the scalar multiplier that could be used to solve the item.

Item	Version A	Version B
Soup	91	51
Curly $\mathcal{L}$	36	75
Dollars	22	27

Roughly correct responses (%)

Item	Version A	Version B
Soup	$\times 3$	$\times 2\frac{3}{11}$
Curly $\mathcal{L}$	$\times 2\frac{5}{8}$	$\times 4$
Dollars	$\times 3\frac{1}{3}$	$\times 4\frac{1}{2}$

Scalar multiplier for each item

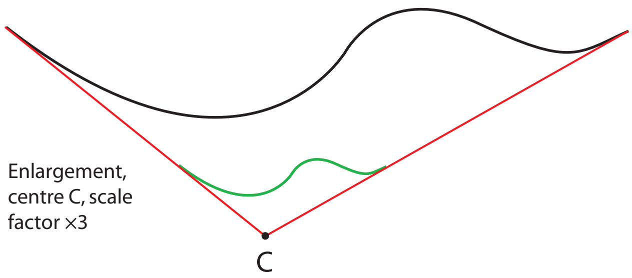
As can be seen, the currency conversion items (Dollars) were the hardest, presumably in large part because the numerical relations are more difficult. On the other hand, the geometric enlargement item (Curly L) was on average harder than the recipe item (Soup), even though the numerical relations are comparable. This suggests that students find it harder to see that the relations are multiplicative in a geometric context than in a recipe context. This is supported by the fact that a sizeable minority of students used the addition strategy (leading to the answer 45) on the Curly L item (22% and 8% respectively for Versions A and B), but very few did so on the Soup item (1% and 3% respectively for Versions A and B). We discuss this context issue further below.

It is also interesting to note that the version of Soup and Curly L with the simpler scalar multiplier turned out to be subtantially easier than the other version (ie with the more complex scalar multiplier but simpler functional multiplier). This suggests that the numerical relations that students construct or focus on is influenced by the context - they don't simply home-in on the simpler relation - and that they seem to prefer scalar to functional relations, eg relations that map people onto people and sauce onto sauce, rather than, say, people onto sauce.

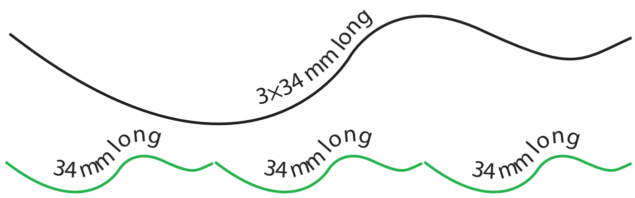
Models of multiplication: scaling

In the 2A and 2B lessons that follow this Starter we look at mutiplication in terms of scaling. One of the first models of multiplication that students learn is repeated addition. This provides a secure basis for multiplying numbers when they are whole (it is difficult to repeat something 2 and a quarter times, say) and when they are small (so that there are not too many additions to repeat). However, it also depends on the context. Repeated addition fits a story like "Every day I eat 3 packs of crisps. How many packs do I eat in 4 days?". But it does not fit "Crisps come in 3 pack sizes and 4 flavours. How many different types of pack are there?".

One context where scaling arises is geometric enlargment, as in the example below, where the small (green) S shape has been enlarged by a scale factor ×3, with centre of enlargement C.



If the small S shape has a curved length of 34 mm, then the enlarged S shape will have a curved length of 3×34 mm. However, it is not helpful to attempt to 'add' three versions of the small S shape together to make the enlarged S shape (below): taken together, the three small S shapes look very different form the enlarged S shape, and it is not at all obvious from the diagram that their combined length is the same as the length of the enlarged S shape. (This kind of demonstration only works when the lines are straight, ie lie in one dimension.) Thus it is more appropriate to think of the larger S shape as resulting from a scaling process rather that an additive process.



It is worth noting that scaling does not apply only to geometric situations. We can scale other quantities, eg wages in China to wages in Japan, and conversions, like (number of) gallons to (number of) litres, pounds to Euros.

In this lessons 2A and 2B we stay with a geometric context by considering the number of metres and feet represented by lines on a map. To keep things simple we restrict ourselves to straight lines, which opens up the possibility again of using additive strategies. However, the straight map-lines serve as a 'natural' introduction to the 'double number line' (DNL) which is an extremely useful (but also challenging) device for representing scaling, and which we consider further in later lessons, including for non-geometric contexts. Our prime interest in the DNL, here and in later lessons, is as a model *of* multiplication, rather than *for* multiplication - ie as a device to help us think about the nature of multiplication, rather than as a device for finding numerical answers.

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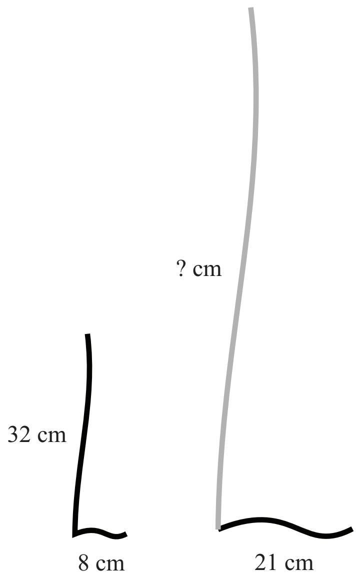

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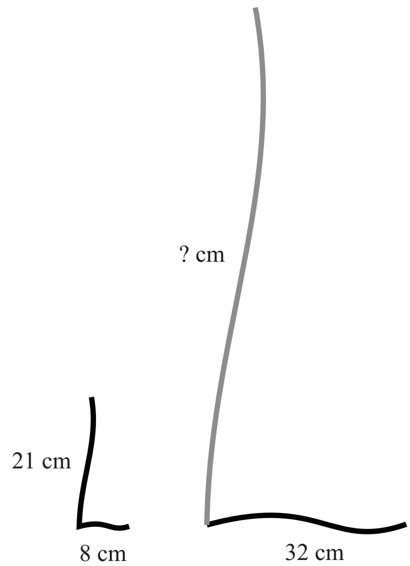

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