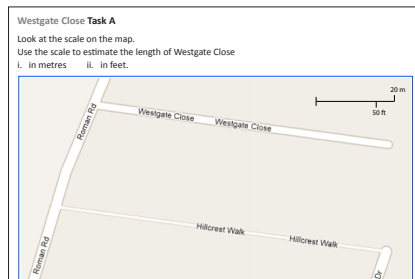# Multiplicative Reasoning: Lesson 2A

Westgate Close

Students estimate the length of Westgate Close, in metres and feet (Task A).
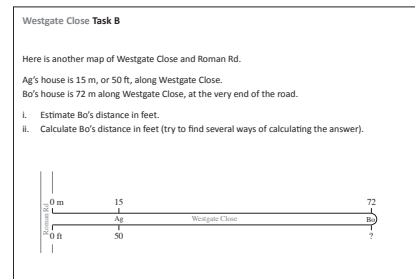
They then estimate and calculate two distances along Westgate Close (Task B and Task C).
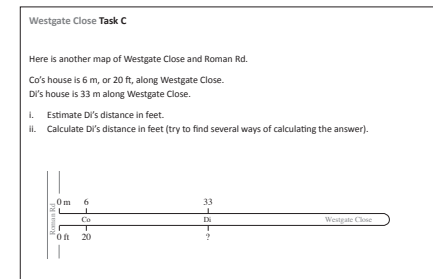
Task A

> **Westgate Close Task A**
> Look at the scale on the map.
> Use the scale to estimate the length of Westgate Close
> i. in metres   ii. in feet.
>
> 20 m
> 50 ft
>
> Roman Rd
> Westgate Close   Westgate Close
> Roman Rd
> Hillcrest Walk   Hillcrest Walk
> Cr

Task B

> **Westgate Close Task B**
>
> Here is another map of Westgate Close and Roman Rd.
>
> Ag's house is 15 m, or 50 ft, along Westgate Close.
> Bo's house is 72 m along Westgate Close, at the very end of the road.
>
> i.   Estimate Bo's distance in feet.
> ii. Calculate Bo's distance in feet (try to find several ways of calculating the answer).
>
> 0 m   15   72
> Ag   Westgate Close   Bo
> 0 ft   50   ?

Task C

> **Westgate Close Task C**
>
> Here is another map of Westgate Close and Roman Rd.
>
> Co's house is 6 m, or 20 ft, along Westgate Close.
> Di's house is 33 m along Westgate Close.
>
> i.   Estimate Di's distance in feet.
> ii. Calculate Di's distance in feet (try to find several ways of calculating the answer).
>
> 0 m   6   33
> Co   Di   Westgate Close
> 0 ft   20   ?

Summary

In this lesson we look at multiplication in terms of *scaling*, and model this with the Double Number Line (DNL).

We use the context of a map, with scales showing distances in metres and feet, to *introduce* the DNL model. At the same time we *use* the DNL model to convert metres to feet, and we relate this to the use of ratio tables.

Outline of the lesson

1. Task A: estimate the length of Westgate Close, in metres and feet.

   - Ask for some quick estimates.
   - Ask students to work on the task in small groups.
   - Briefly discuss their methods and results.

2. Task B: change a 72 m distance on a map to feet.

   - Ask students to work on the *estimate* in small groups.
   - Briefly discuss their methods and results.
   - Ask students to work on the *calculation* in small groups.
   - Discuss their methods and results.

3. Task B: change 72 m to feet using a ratio table.

   - Represent the information in Task B in a ratio table. Discuss ways of operating on the numbers to find the required value in feet.

15	72
50	•

4. Task C: change a 33 m distance on a map to feet.

   - Ask for some quick estimates.
   - Represent the information in Task C in a ratio table. Ask students to work on the *calculation* in small groups, using the map (DNL) and/or the ratio table.
   - Discuss their methods and results.

6	33
20	•

5. Possible extension: revise Task C.

   - Ask students to change the given distance, 33 m, to a different distance, to make the task easier or harder.

6. Possible extension, Task D: assess students' work.
   - Ask students to evaluate some responses to Task B.

Overview

Mathematical ideas

In this lesson we use a map to introduce students to the Double Number Line (DNL), and we use this and ratio tables to explore conversions between measurements involving metres and feet.

The tasks in this lesson are similar to the Dollars conversion item on the *Mini Ratio Test* Starter, and the relations between the given numbers (eg $15 \times 3\frac{1}{3} = 50$) are at a similar level of complexity. This suggests that some students might well construe the situation as additive – eg they might conclude that 72 m is the same as 107 ft, because 50 is 35 more than 15, and 72 plus 35 is 107. This lesson is designed to help students see that the situation is multiplicative and introduces them to a multiplicative model in the form of the DNL.

Students' mathematical experiences

Students

- estimate lengths

- talk about ratio problems and how to solve them

- use the DNL and ratio tables to model problems involving ratio.

Key questions

How did you work that out?

We've got two answers and an explanation for both. How can we decide between them?

Assessment and feedback

The *Mini Ratio Test*, in particular students' responses to the Dollars conversion item, should help you anticipate the kinds of methods that will arise in the lesson - eg addition, rated addition, scaling, the unitary method. Find space in the lesson to make such methods explicit but don't at this stage feel that you have to try to resolve all the misconceptions that might come up.

Task D gives students an opportunity to reflect on different methods and ideas. It involves assessing work done by other students and thus provides practice in peer- and self-assessment.

You might also want to give students similar tasks to the ones in the lesson but involving a different ratio context - a recipe, for example. Do students find this easier or harder?

Adapting the lesson

The extension activity in Stage 5 asks students to revise Task C by choosing numbers that would make it easier or harder. This is a useful way for students to reflect on the nature of a task and methods of solution, and it provides insight for the teacher into the nature of students' understanding. You might want to adopt this strategy yourself, earlier in the lesson, if the given numbers prove to be too easy or hard.

You might also, here or in a later lesson, want to discuss how the DNL and ratio table can be used to model and solve the items in the *Mini Ratio Test*.

Outline of the lesson (annotated)

1. Task A: estimate the length of Westgate Close, in metres and feet.

    - Ask for some quick estimates.
    - Ask students to work on the task in small groups.
    - Briefly discuss their methods and results.

    — This can be done quite effectively by eye, by marking off distances of approximately 15 m and distances of approximately 50 ft.

2. Task B: change a 72 m distance on a map to feet.

    - Ask students to work on the *estimate* in small groups.
    - Briefly discuss their methods and results.

    — Again, this can be done by eye, or by using the idea that there are '50 ft for every 15 m', and that that 5 × 15 (or 15+15+15+15+15) is a little over 72: so the distance in feet will be a bit less than 5 × 50 (or 50+50+50+50+50).

    - Ask students to work on the *calculation* in small groups.
    - Discuss their methods and results.

    — It is likely that some students will use the 'addition strategy' here, and thus get an answer of 107 ft rather than 240 ft (by arguing "15+57=72, so 50+57=107", or "15+35=50, so 72+35=107").

    Ask the class to critique this. For example,
    - How does this answer compare to the earlier estimates?
    - How does it fit with the idea of '50 ft for every 15 m'?
    - How may ft would the method give for 16 m, or 1 m?

3. Task B: change 72 m to feet using a ratio table.

    - Represent the information in Task B in a ratio table. Discuss ways of operating on the numbers to find the required value in feet.

    — It is possible to find the desired number of feet (240) by using the multiplier ×3.3̇, which maps (the number of) metres onto (the number of) feet. Thus 15×3.3̇ = 50, and 72×3.3̇ = 240.

    It is also possible to use the multiplier ×4.8, which maps 15 m onto 72 m, and thus maps 50 ft onto 50 ft × 4.8 = 240 ft. Challenge students to find these multipliers, if they don't arise spontaneously.
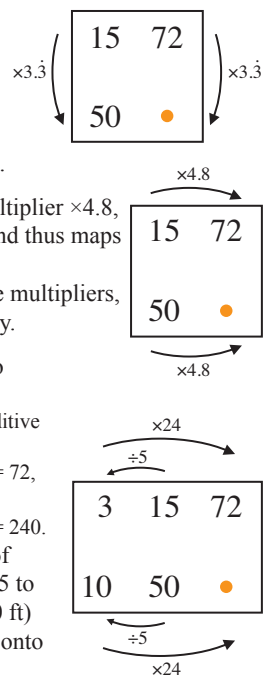

    However, first allow students to report on their own ideas. These might include successful additive strategies, eg 'rated adition':
    15 + 15 + 15 + 15 + 15 − 1/5 of 15 = 72, so similarly
    50 + 50 + 50 + 50 + 50 − 1/5 of 50 = 240.
    Or they might include the use of intermediate values, eg using ÷5 to map (15 m, 50 ft) onto (3 m, 10 ft) and then using ×24 to map this onto (72 m, 240 ft).

4. Task C: change a 33 m distance on a map to feet.

    - Ask for some quick estimates.
    - Represent the information in Task C in a ratio table. Ask students to work on the *calculation* in small groups, using the map (DNL) and/or the ratio table.
    - Discuss their methods and results.

    — Try to make links between students' use of the double number line and students' use of the ratio table.

5. Possible extension: revise Task C.

    - Ask students to change the given distance, 33 m, to a different distance, to make the task easier or harder.

6. Possible extension, Task D: assess students' work.

    - Ask students to evaluate some responses to Task B.

   Ask students to evaluate these responses (see Task D). This could be set for homework.

Background

### Scaling in the context of conversions and enlargement

Though Westgate Close involves distances on a map, it does not involve geometric enlargement. We are simply converting a distance on a map measured in one unit to the same distance measured in another unit. We are not scaling a distance on a map to the equivalent distance in real life or onto a map drawn to a different scale.

Thus the Westgate Close tasks are probably closer to the conversion items (Dollars) on the *Mini Ratio Test* than to the enlargement items (Curly L). As such, they are probably less demanding than the Curly K task shown below, where the scalar and functional multipliers are fractional ($\times 1\frac{1}{2}$ and $\times 1\frac{1}{8}$, respectively), as with the Westgate Close tasks. The item is from the CSMS Ratio test (Hart, 1981), and was answered correctly by just 14 % of a representative sample (N=309) of Year 8 students in 1976, and by a similar proportion (12%) of a representative sample (N=754) of Year 8 students in 2008.

These 2 letters are the same shape. One is larger than the other. AC is 8 units. RT is 12 units.

The curve AB is 9 units. How long is the curve RS ? . . . .
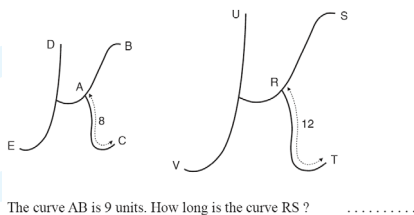
One possible reason why the success rate on this item is so low is that the addition strategy response of 13 is quite close to the correct answer, 13.5. This is not so for Westgate Close. The response below (for an early version of Task B) was given by a student from a low attaining Year 8 group. He has not made any attempt to relate metres and feet but his estimate of 220ft for the length of Westgate Close is near to the correct distance (240ft) and is very different from the distance (107ft) produced by the addition strategy. Thus students who use the addition straegy here have a strong reason to think again.
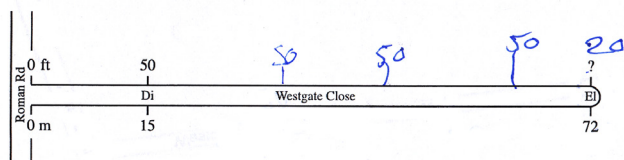
The work below is by a student from a very high attaining Year 8 class. She first came up with the addition strategy answer 107 (from 72+35 and perhaps 50+57), but
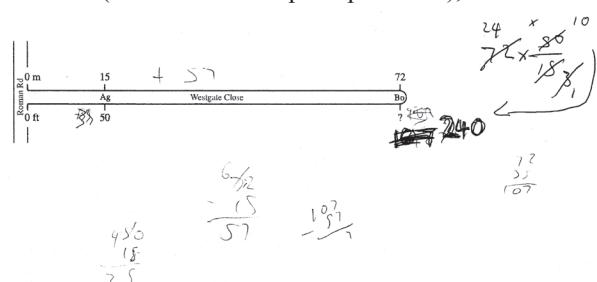
subsequently arrived at the correct answer, 240, probably in the light of a whole-class discussion.

Note: In this lesson we are using the conversion "15 m corresponds to 50 feet". This is not exact - the actual relation is more complex.

Ratio tables

A table can be very useful for organising information in a problem involving ratio, ie for showing how the value one is trying to find corresponds to the given information. It also provides a useful means for reflecting on and recording the operations that one might perform to find the missing value. Ratio tables are particularly easy to create for the Westgate Close tasks, since we can use a similar layout to the way the numbers are presented in the given schematic maps.
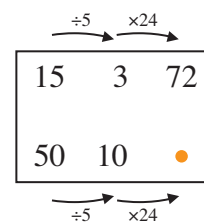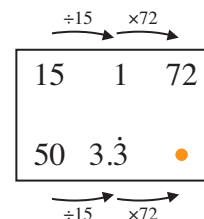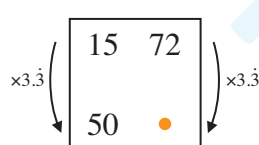
Addition strategy

Rated addit-ion

$$15+15+15+15+15-3 = 72$$
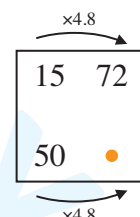$$50+50+50+50+50-10 = \bullet$$
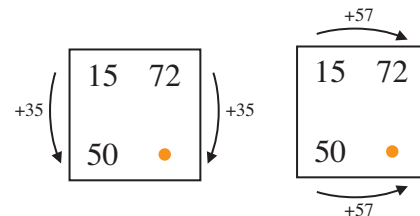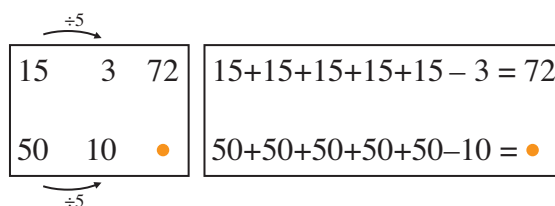
Use of a rate ("10 for every 3")
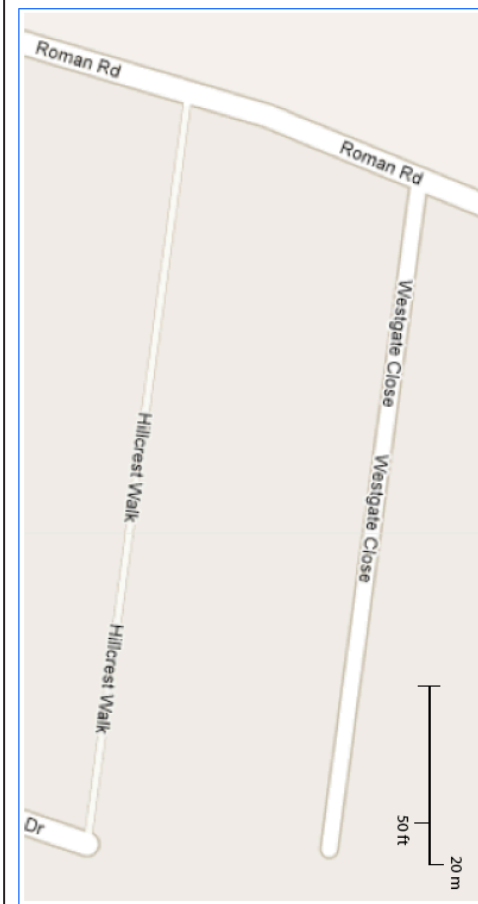
Unitary method

Functional multiplier

Scalar multiplier

Westgate Close Task A

Look at the scale on the map.

Use the scale to estimate the length of Westgate Close
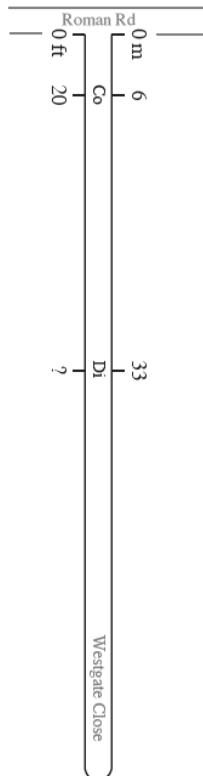
i. in metres   ii. in feet.

Westgate Close Task B

Here is another map of Westgate Close and Roman Rd.

Ag's house is 15 m, or 50 ft, along Westgate Close.

Bo's house is 72 m along Westgate Close, at the very end of the road.

i. Estimate Bo's distance in feet.

ii. Calculate Bo's distance in feet (try to find several ways of calculating the answer).
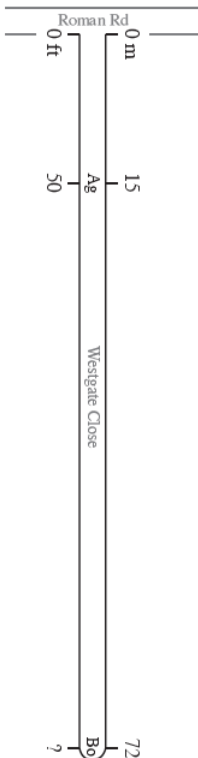
Westgate Close Task C

Here is another map of Westgate Close and Roman Rd.

Co's house is 6 m, or 20 ft, along Westgate Close.

Di's house is 33 m along Westgate Close.

i. Estimate Di's distance in feet.

ii. Calculate Di's distance in feet (try to find several ways of calculating the answer).

Westgate Close Task D: Assessing students' work

Here are three students' answers to Task B.

1. Look carefully at their work. Try to explain the method you think they have used.

2. Write a helpful comment for each student.

Student A

$$\begin{array}{r} \times\ 4 \\ +\ 12 \\ \hline 212 \end{array}$$

Student B

$$4.8 \times \quad 15 : 50 \quad \times 4.8 \\ 72 : 240$$

Student C

$$50 + 15 = 35 \qquad 72 + 35 = 107\ ?$$