Algebra: Lesson 1B

Comparing expressions

Which is bigger, $3n$ or $n + 3$?

Summary

The students compare two algebraic expressions as in Boat Hire. However, the problem is set in a 'pure' mathematical context, and some students may find this more difficult.

As in Boat Hire a variety of representations are used to compare the two expressions. You should place more emphasis on continuity and extending n to include rational and negative numbers.

Outline of the lesson

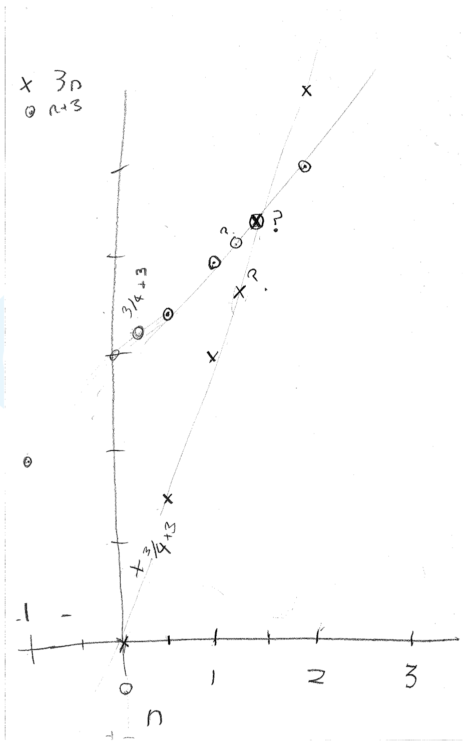
1. Display the problem, and, if necessary, remind the students that they have looked at this problem before.
    - Discuss which of the two expressions is bigger: $3n$ or $n + 3$, allowing time and space for students to express different ideas.
    - Prompt alternatives, if necessary: Does anyone think that $n + 3$ is always bigger?
    - Encourage correct and incorrect explanations, recording them on the board.
    - Ask students to consider the problem further in pairs.

2. Collect numerical data on the board. As in the Boat Hire lesson, record these initially in a random layout.
    - Encourage the students themselves to suggest using a table, then an ordered table of values.
    - Can $3n$ and $n + 3$ ever be the same?

3. Put a standard pair of Cartesian axes on the board.
    - Ask students to represent some of the values they've come up with by placing a few of the points on the axes.
    - You may want to ask students, in pairs, to construct their own graphs
    - Can we draw a line through the points?

4. Discuss and make links between the various representations and between them and the algebra.
    - Ask students to use the graph to predict pairs of numbers that satisfy each relation, then check, recording these in the table.
    - Ask: Can you use the algebraic relations to predict points on each graph? Are you right?

5. Homework: Investigate similar problems.
    - Say to students: Make up your own "Which is bigger?" problem and show how it could be solved.



Note: These materials are the subject of ongoing research and are made available on request to teachers as draft trial materials only. Please send feedback to Jeremy.Hodgen@kcl.ac.uk or Dietmar.Kuchemann@kcl.ac.uk

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Overview

Mathematical ideas

A straight-line graph, such as $y = 3n$ or $y = n + 3$, is a set of points that all satisfy a particular relation.

The different tools used in this lesson (Cartesian graphs, symbolic algebra, tables of values and everyday language) are different ways of representing the same algebraic relationships.

Students' mathematical experiences

Allow students plenty of time to make, discuss and correct mistakes, such as abbreviating $n + 3$ to $n3$.

The students should …

- revisit and extend the mathematical ideas they encountered in Boat Hire
- justify, discuss and check their ideas
- investigate whether points on the Cartesian graph satisfy the relation expressed in symbolic algebra or in everyday language, and vice versa
- discuss the differences (and similarities) between $3n$, $n3$ and $n + 3$ in everyday language and in symbolic algebra.

The students should discover …

- sometimes $3n$ is greater and sometimes $n + 3$ is greater, depending on the value of n
- when $n = 1.5$, the two expressions are equal
- as n increases by 1, then $3n$ increase by 3, but $n + 3$ only increases by 1.

Key questions

Why do you think that …?

What do you notice about the graph?

Is there a point where $3n$ is equal to $n + 3$?

Can you predict what would happen for $n = {}^{1}/_{2}$, $n = {}^{-}1$, …?

Assessment and feedback

This lesson revisits the starter activity to this pair of lessons. Look back at your notes to decide what areas to focus on.

Choose 3 students to assess during the lesson. Talk to each at least once during the lesson and observe them working.

Some students may argue that

- $3n$ is greater, because "multiplication always makes things bigger"
- $n + 3$ can be abbreviated to $n3$ or $3n$ and, hence, that the two expressions are exactly the same.

Encourage all these ideas and allow time for students to explore them using the different representations.

The relationships can be extended to fractions, decimals and negative numbers. However, some students may have difficulty multiplying (or adding) such numbers. Discuss the different methods that students use, as a class.

Adapting the lesson

Think up other expressions for students to compare (or select expressions that students have come up with). Keep them fairly simple. You might want to control for what kind of value of the unknown (small whole number, large whole number, fraction?) the values of the expressions coincide. And you might want to vary the operations (eg compare $x+14$ and $100–x$, or $x+14$ and $100÷x$).

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Outline of the lesson (annotated)

1. Display the problem, and, if necessary, remind the students that they have looked at this problem before.
   - Discuss which of the two expressions is bigger: $3n$ or $n + 3$, allowing time and space for students to express different ideas.
   - Prompt alternatives, if necessary: Does anyone think that
     $n + 3$ is always bigger?
   - Encourage correct and incorrect explanations, recording them on the board.
   - Ask students to consider the problem further in pairs.

The problem is related to this question from the CSMS Algebra test: Which is larger, $2n$ or $n + 2$? Nationally, only 4% of Y8 students answered this item correctly. However, the task is easier when worked on collectively, in a classroom setting.

2. Collect numerical data on the board. As in the Boat Hire lesson, record these initially in a random layout.
   - Encourage the students themselves to suggest using a table, then an ordered table of values.
   - Can $3n$ and $n + 3$ ever be the same?

Some students may not immediately recognize $3n$ as $3 \times n$. Encourage the students to clarify the syntax issue for themselves by talking about the expressions.

3. Put a standard pair of Cartesian axes on the board.
   - Ask students to represent some of the values they've come up with by placing a few of the points on the axes.
   - You may want to ask students, in pairs, to construct their own graphs
   - Can we draw a line through the points?

The expressions are equal when $n = 1.5$. Students will need some time to consider and discuss the multiplication (and addition) of fractions and decimals. Encourage class sharing and discussion of the students' ideas and methods - resist simply reminding them of previously encountered formal methods.

4. Discuss and make links between the various representations and between them and the algebra.
   - Ask students to use the graph to predict pairs of numbers that satisfy each relation, then check, recording these in the table.
   - Ask students to use the algebraic relations to predict Ask: Can you use the algebraic relations to predict points on each graph? Are you right?

Allow students plenty of time to explore whether the points on the straight line do actually satisfy the relation.

5. Homework: Investigate similar problems.
   - Say to students: Make up your own "Which is bigger?" problem and show how it could be solved.

The students themselves are likely to suggest expressions of the form kn and $n + k$, $4n$ and $n + 4$, $5n$ and $n + 5$ etc. You may want to suggest expressions involving different letters or large numbers such as $4x$ and $x + 100$. Prepare these in advance and then adapt / use as appropriate during the lesson.

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Background

Assessment

This lesson is an ideal opportunity for formative assessment and feedback. However, giving thoughtful feedback to the whole class can be very time consuming. One way of dealing with this is to assess the work of a few students in depth.

Look at the students' classwork together with this homework task: Make up your own "Which is bigger problem" and show how it could be solved.

Consider the issues raised for teaching the next lessons. For example:

- How do the students understand the key ideas of variable and the use of algebraic symbols and a Cartesian graph?
- Are there any related issues in their understanding [such as the multiplication of fractions]?

Comments to students could include one thing they have understood and one thing they need to do. A possible comment for student F in the video might be:

> It was good that you noticed that the points for 3*n* and *n* + 3 seem to lie in straight lines. Can you test this with some other values for *n* ? … … 2 … 1.4?
>
> Using a number line to work out the fraction multiplications was a good idea. Compare your answers with someone else too.

An interview with 3 students

We have a lengthy video recording of an interview with three Year 8 students on this task.

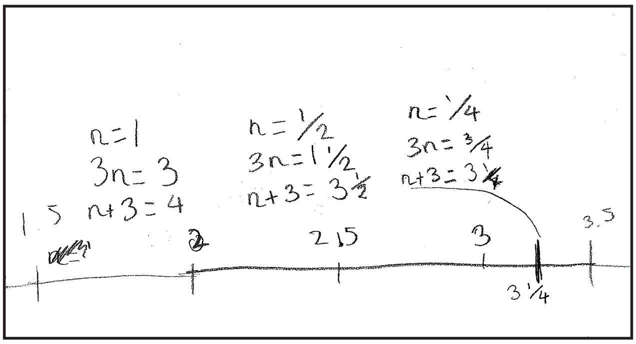
You could watch the following extracts:

Extract A [28.00-34.18]: One student makes a generalization involving whole numbers ("3*n* is bigger if *n* equals more than 1, like 2, 3, 4, 5 and 6, and *n* + 3 is bigger if the number's less than 1, like 1, a half, a quarter and an 8th, and 3*n* and *n* + 3 are the same if *n* equals infinity") and they begin using Cartesian axes.

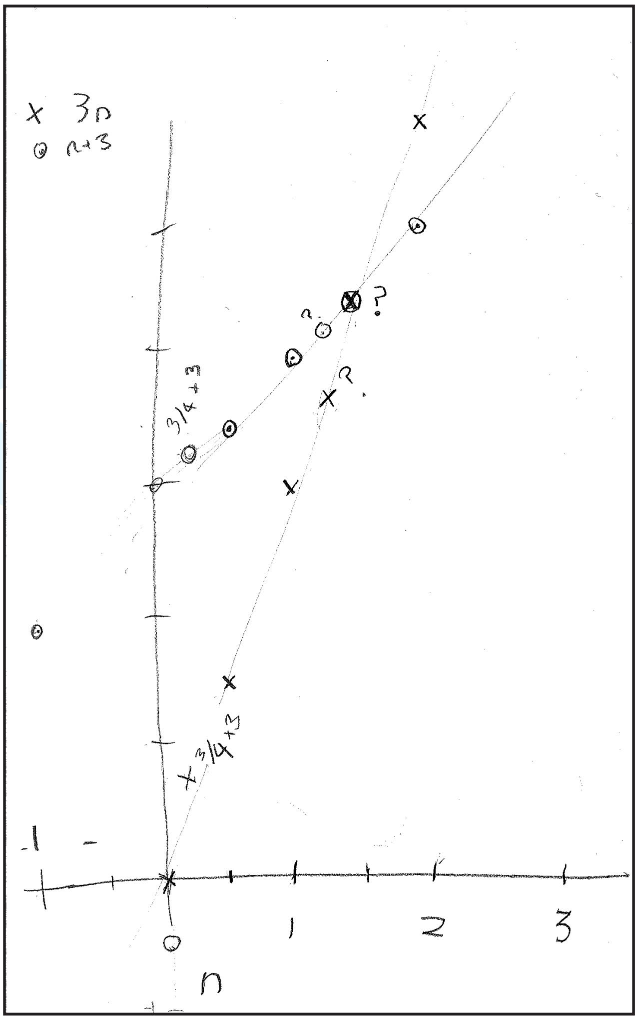
Extract B [35.20-40.20]: They make predictions, suggest that the points seem to lie in a line and consider the values of 3*n* and *n* + 3 when *n* = 0.

The extracts can be viewed from the project website. Go to www.iccams-maths.org

The figure below shows where the students have listed values of the expression for *n*=1, *n*=½ and *n*=¼, and where they have marked one of the values on a number line.



On the page below, the axes and some of the points were drawn by one of the interviewers. The lines through the points were drawn, in a very confident and fluent way, by one of the students, who also added further points.



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