Textbooks for the teaching of algebra in lower secondary school: are they informed by research?

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Over the past 30 years, there has been a great deal of work directed at, first, understanding students' difficulties in algebra and, second, examining ways of tackling these difficulties. There is a vast research literature detailing successful ways of teaching algebra in experimental and laboratory settings. Yet, there is no clear evidence that this work has had a significant effect in terms of improving either attainment or engagement in algebra in ordinary, or non-experimental, classroom settings. Indeed, in England, current attainment in algebra appears to be no better than that of 30 years ago. In this paper, we analyse the algebra topics from two textbooks currently in widespread use in England, focusing on Grade 7 (ages 12–13). We examine the extent to which these textbooks draw on the research literature to support the teaching of algebra. Finally, we discuss the implications of this study.

Keywords: mathematics; textbooks; algebra

Introduction

Some 40 years ago, in 1969, the first International Congress of Mathematics Education was held in Lyon, France. *Educational Studies in Mathematics*, the first international journal dedicated exclusively to mathematics education research, had been established the previous year. In the intervening period there has been an immense amount of research on the teaching and learning of algebra. Yet, there is no clear evidence that this research has had a significant effect in terms of improving either attainment or engagement in mathematics. Indeed, the teaching and learning of algebra continues to be a major policy concern around the world (e.g., National Mathematics Advisory Panel, 2008).

The research we report here is drawn from the *Increasing Competence and Confidence in Algebra and Multiplicative Structures* (ICCAMS) study in England. As part of this study, we have compared students' understandings of algebra currently with that of the 1970s and have found that the understandings of 14-year-old students are no better now than 30 years ago (Hodgen, Küchemann, Brown, & Coe, 2009). In seeking to understand this phenomenon, one would ideally compare mathematics classrooms then and now. Unfortunately, there is little past data or evidence that provides a direct comparison of practices in school algebra of the 1970s with those of today. However, it is possible to examine secondary sources, such as curriculum materials, policy documents, official reports, and examination papers. Mason and Sutherland's (2002) analysis suggests that concerns about school algebra have changed little over time. They observe that there is a remarkable consistency in official reports going back to the beginning of the last century and before.

Official reports certainly provide insights into school algebra, but they are constructed at some distance from the classroom. An alternative approach is to focus on textbooks. Textbooks are almost ubiquitous in mathematics classrooms across the developed world and are amongst the most influential factors in the implemented curriculum. In the 2007 Trends in International Mathematics and Science Study (TIMSS) survey, for example, 96% of teachers at Grade 8 internationally reported that they made at least some use of textbooks to teach mathematics (Mullis et al., 2008).[1]

In this paper, we compare the presentation of algebra topics in the Year 8 (Grade 7) textbooks from two best-selling series: one from the 1970s, the *School Mathematics Project (SMP) Letter Series* published over the period 1968–1972; and one currently in use, *Framework Maths* published over the period 2002–2004. Our aim here is to examine what substantive changes have been made to the textbooks over the intervening period and, in particular, the extent to which the research basis on the teaching and learning of algebra has impacted on the current textbook.

The research on mathematics textbooks

Whilst there is a great deal of research investigating the implementation of curriculum resources, the research on textbooks is much more limited. The 1992 *Handbook of Research on Mathematics Teaching and Learning* (Grouws, 1992) has no chapters on textbooks or curriculum materials, whilst the second edition of the *Handbook* contains only one chapter (Stein, Remillard, & Smith, 2007).

One strand of textbook research compares textbook use internationally. Such comparisons can be useful, but as Foxman (1999) points out in his review of TIMSS, they need to be treated with caution since textbooks provide only a partial picture of classroom practice. Similarly, Howson and Robitaille (1995) argue that such comparisons "give indications…[but] are not determinants of national characteristics or necessarily of classroom practice" (p. 13). In their study, comparing textbooks in three European systems, Haggarty and Pepin (2001) highlight the issue of mathematical complexity and coherence. They found English textbooks to be less coherent and more routine than the French and German ones. Other researchers investigate textbook use within a particular national system. For example, Gu, Huang and Marton's (2004) research on Chinese textbooks highlights the importance of variation, particularly in the use of examples and non-examples. In an examination of Hong Kong and Korean teaching, Leung and Park

(2002) illustrate how teacher's manuals in both countries provide a coherent and consistent mathematical basis for teaching, albeit largely in the context of procedural teaching.

Another approach examines how teachers actually use textbooks. Askew (1996), for example, examines the extent to which teachers actively seek to interpret and modify the textbook. He highlights a distinction between scheme-driven and scheme-assisted approaches. Similarly, Brown (2008) distinguishes between teachers offloading, adapting and improvising. Remillard and Bryans (2004) suggest that as teachers' expertise develops, they are more able to adapt and modify curriculum materials. Stein and Kim (2008) argue that, alongside providing "steps to follow, problems to give [and] actual questions to ask", textbooks need to engage teachers "in the rationales, assumptions or agendas that undergird these actions" (p. 44). Further, they argue that materials should help teachers anticipate the range of students' potential responses.

Background and context

The ICCAMS study

The ICCAMS is a 4-year research project funded by the Economic and Social Research Council as part of a wider initiative aimed at identifying ways to improve participation in Science, Technology, Engineering and Mathematics (STEM) disciplines in the UK.

Phase 1 of the project consists of a large-scale survey of 11–14-year-olds' understandings of algebra and multiplicative reasoning in England. A representative sample of approximately 6,000 students has been surveyed across 2008 and 2009 using tests originally used in the 1970s as part of the *Concepts in Secondary Mathematics and Science* (CSMS) study (Hart, 1981). Phase 2 consists of a collaborative research study with teacher-researchers who are using the results of Phase 1 on their own Year 8[2] classes, as part of extending the investigation to classroom/group settings. One aspect of this study is to examine how to support teachers in using textbooks to improve student attainment and attitudes.

School mathematics in England

Unsurprisingly, school mathematics education in England has undergone change in the past 30 years since the original CSMS study. Indeed, many of these changes make England a particularly interesting case of school mathematics. Of note are several national initiatives directed at systemic improvement of mathematics teaching and attainment: a National Curriculum; national testing at ages 7, 11 and 14; the National Numeracy Strategy; and the Secondary National Strategy. The national initiatives are of particular interest in that they were directed at improving classroom pedagogy, and introduced guidance on lesson structure together with a very detailed outline and exemplification of the mathematics curriculum—the *Frameworks for Teaching Mathematics* (Department for Children, Schools and Families [DCSF], 2008; Department for Education and Employment [DfEE], 2001). Many of these initiatives have drawn directly or indirectly on the results of the CSMS study (Brown, 1996). During this period, examination results in England have shown steady and substantial rises: for example, the proportion of students achieving grade C or above in the GCSE[3] has risen from 45% in 1992 to 57% in 2009.

However, independent measures of attainment suggest that these rises may be due more to "teaching to the test" than to increases in genuine mathematical understanding. Initial results from Phase 1 of the ICCAMS study suggest that, whilst students' understandings of algebra at Year 8 (age 13) are generally better than in 1976, at Year 9 (age 14) their understandings are broadly similar to those of 1976 (Hodgen et al., 2009).[4] Replication results from the science strand of the CSMS study show a similar pattern, that students' understanding of some algebraic ideas has not risen and has in some respects declined (Shayer & Ginsburg, 2009).

Moreover, despite these increases in examination performance, students' attitudes to mathematics do not appear to have improved. Insufficient students are choosing to continue studying mathematics post-16. There is considerable research in the UK addressing reasons for non-participation in mathematics—students stop studying mathematics because their experience of it is difficult, abstract, boring and irrelevant (Osborne et al., 1997). This commonly relates to algebra and the predominance of routine and formal work within algebra.

Although lower than the international average, textbook use in English secondary classrooms is high, with Grade 8 teachers reporting that they make some use of textbooks to teach mathematics in 89% of lessons (Mullis et al., 2008). The figure for the use of textbooks as the principal basis for mathematics lessons is 43% compared to the international average of 60%. This lower figure may reflect a greater use of Internet resources and initiatives by the Secondary National Strategy promoting the use of alternative resources alongside published textbooks. It may also reflect a view amongst the educational establishment that schools over-rely on textbooks rather than undertaking their own detailed planning. Our own and colleagues' observations suggest that, whilst teachers often use alternative resources at the beginning of lessons, textbooks still provide the principal source of the problems set for students in class. Textbooks in England are commercially produced and, in contrast to some other national systems, there is no process of state approval. Hence, schools and teachers are free to choose whatever textbooks they wish to use (see Table 10 “Control and supply of school textbooks”, International Review of Curriculum and Assessment Frameworks Internet Archive, 2009). Since the introduction of the National Curriculum in 1989, the educational system in England has been subject to very frequent reform and review. One consequence of this has been to limit the time available for the development and trialling of textbooks.

Methods

The analysis reported here is part of an ongoing aspect of the ICCAMS study. This aspect of the research is directed at understanding the ways in which current textbooks support the teaching of algebra and at identifying ways in which such textbooks might be used to support more effective teaching and learning.

The first task was to identify textbooks that represent mathematics textbooks in use currently and in the 1970s. In the case of current textbooks, we were able to easily identify the two market-leading textbooks for students aged 11–14: *Framework Maths* published by Oxford University Press, and *Maths Frameworking* published by Collins Education. The use of the terms “framework” and “frameworking” refer to the *Framework for Teaching Mathematics* (DCSF, 2008; DfEE, 2001). The prominence of these terms attests to the importance attached by schools and government to this curriculum guidance. For reasons of space, in this article, we focus on just one of these, *Framework Maths*—the series that we judge to be the currently best-selling and most widely used. There are no definitive publicly available statistics for textbook sales, and the publishers of both *Framework Maths* and *Maths Frameworking* claim their series to be the market leader. Our judgement is based on the sales ranking on Amazon (UK), our own survey of schools, and our own anecdotal knowledge of school mathematics. Additionally, our analysis suggests that *Framework Maths* shows considerably noticeably more evidence, than *Maths Frameworking*, of attempts both to present a coherent picture of mathematics and to incorporate aspects of research.

For the 1970s, the choice of textbooks series was surprisingly straightforward in that there is strong evidence from contemporaneous and historical studies of a clear market leader: the *SMP Letter Series: A–H* and *X–Z* (Anderson, 1978; Breakell, 2001; Thwaites, 1972). A word of caution is necessary at this point in that there was a greater variety in the style and content of mathematics textbooks in the 1970s. In addition to its widespread use, the *SMP Letter Series* was aimed at providing a course for students across the attainment range.

We analysed the textbooks as follows. First, we compared the broad structure and content covered across all the books and the associated teacher's guides. This demonstrated, as we ourselves had anticipated, that currently formal algebra is taught earlier than it was in the 1970s, that more formal algebra is taught in lower secondary school in England today, and that the sub-divisions of content are rather different. This process also enabled us to identify the most directly comparable chapters or units in terms of content: namely, the unit in which linear relations, or straight line graphs, are introduced. Second, each of the authors of this paper wrote extended prose analyses of the various textbooks with a particular focus on the chapters on linear relations. Finally, we collated these analyses into a structured form, comparing the textbooks on the issues arising from research discussed below.

Identifying significant issues from the research on algebra

There is an enormous body of research in mathematics education that is relevant to the teaching and learning of algebra, much of it published since the publication of the *SMP Letter Series*. Our review of the literature was informed by the research on textbooks summarized above, greatly aided by two recent international reviews with a similar purpose conducted in England (Mason & Sutherland, 2002; Watson, 2009) and an influential report by the Royal Society (1997). Through this process, we identified the following areas, which we believe are well-known and well-developed in the literature.

Mathematical coherence

By mathematical coherence, we refer to the extent to which the mathematics content is presented in a coherent and connected manner, and the extent to which the text illuminates the mathematical ideas and concepts, an issue that arises directly from the textbook literature (e.g., Haggarty & Pepin, 2001). In our analysis of the chapters on linear relations, we were particularly concerned with the notion of *variable* and the *use of symbols* (Küchemann, 1981). A central idea is that "letters in algebra may apply to *any number* [emphasis added]—not necessarily to some particular unknown number which will solve some particular problem" (The Mathematical Association, 1933, as cited in Mason, Graham, Pimm, & Gowar, 1985, p. 90). In examining this issue, we are concerned with the ways in which the textbook addresses issues relating to the psychological aspects of algebra learning rather than simply the logic of the mathematical ideas (Orton et al., 1995).

Misconceptions

We note that misconceptions, although "incorrect", are not simply mistakes or misunderstandings, but rather have some logical foundation. In the context of this analysis, we have identified the misconception of *letter as object* (rather than as representing generalized number or variable) as of key importance (Hart, 1981). Of similar importance is the notion of *lack of closure*. In developing an understanding of linear functions such as $y = 3x + 2$, students have to be able to see the expression $3x + 2$ as an "answer" and a number in its own right rather than solely as an instruction to do something. At the same time, they have to appreciate that it cannot be re-written as a closed expression like $5x$. One function of textbooks is to provide information that enables teachers to anticipate and plan for such misconceptions (Stein & Kim, 2008).

Multiple representations

This refers to the importance of examining and comparing the many different ways in which algebraic ideas can be represented, for example, graphically, symbolically, numerically, pictorially, in tabular form, and in words (Kaput, 1989). Such representations are important in their own right as well as enabling students to enrich their mathematical understanding.

Variation

Here, we examine the variety of examples chosen and the use of non-examples (e.g., non-linear functions such as $y = x^2$) (see Gu et al., 2004). We also analyse the extent to which the textbooks engage teachers in the rationale for these choices (Stein & Kim, 2008).

Purpose

We examine the extent to which textbooks attend to students perceiving a need for algebra, and thus the opportunity to understand the purpose and utility of these ideas (Arcavi, 1994; Ainley, Pratt, & Hansen, 2006).

Technology use

There is a substantial literature base on how new technology may be used effectively in the teaching of algebra (e.g., Sutherland, Rojano, Bell, & Lins, 2000). We note, however, that the SMP textbook was published before such technologies were available in schools.

Analysing the textbooks

In this section, we first analyse *Book C* from the *SMP Letter Series* (1969), then the Year 8 Core book from the *Framework Maths* series (Capewell et al., 2003b). In both cases, we examine the Teacher's Guide, which for both embeds the student's textbook within its text. Our focus is on the introduction of straight line graphs—the chapter or unit that we identified as the most similar in terms of content and thus most directly comparable: Chapter 3 of *SMP Book C*, "From relation to graph" (pp. 88–115), and Unit A3 of *Framework Maths*, "Functions and graphs" (pp. 85–98). We note that, in each case, the general approach outlined in the chapter is typical of that of the book as a whole.

SMP Book C

The *SMP Letter Series* was intended as a modern mathematics course intended to cover all the mathematics to be taught in secondary school up to the age of 16 (Anderson, 1978). The course was targeted at the new comprehensive state-schools and was aimed at the complete attainment range by providing two overlapping courses: one for the highest attaining 25% (*A*–*G* followed by *X*–*Z*)[5], and one for the remaining 75% (*A*–*H*).[6] Each book provided roughly half a year's work. *Book C* was aimed at the first half of Year 8. The Teacher's Guide consists of the student textbook interspersed with a commentary (with answers) aimed at the teacher on separate pages alongside, mainly discussing the mathematics ideas being taught with links to previous and future chapters.

Chapter 3, "From relation to graph", is intended to cover roughly 2 weeks' work or 8–10 lessons, although the decision of timing and pace is left to the teacher. The chapter is divided into four sections, each with a commentary and pupil exercises: *Representing relations*, *Finding relations*, *Graphs of relations* and *Further relations*.

Mathematical coherence

Representing relations begins by discussing just one relation ($x \rightarrow 3x$) in great depth. The student textbook devotes 5 pages to this, with an additional 2.5 pages in the Teacher's Guide. The Teacher's Guide begins as follows:

> In Book B, Chapter 7, Relations, we studied relations between sets of numbers, representing them as arrow diagrams and ordered pairs and plotting the ordered pairs as Cartesian coordinates. This chapter takes up and *develops the idea of representing relations by means of diagrams*.
>
> However, *there is a considerable step to take in moving from relation to graph*. In Book B we considered relations between finite sets of numbers by a general statement, for example, $x \rightarrow 3x$.…
>
> *We should now like* x *to be a continuous variable instead of standing for an element of a collection of discrete values*. For example, in drawing the graph of $y = 3x$ or $x \rightarrow 3x$ (where x is a real number), *we want to join up the points and attach meaning to every point on the straight line*.…
>
> *Though we do not consider irrational numbers at this stage, they should, nevertheless, be at the back of one's mind in teaching this material*. In plotting the graph of a relation like $y = 3x$, we usually choose rational values of x (that is fractions, like 5/7) which automatically give rise to rational y. If x is irrational (for example, $\sqrt{7}$) then rational approximations to x correspond to rational approximations to $y = 3x$ (in other words the relations is a continuous function). *The fact that the line can be "filled in" is an intuitive description of this fact*. (p. 86, emphases added)

A striking feature of this introduction is that the mathematics to be taught is introduced (to the teacher) in a rigorous and coherent manner, by first discussing how the chapter develops previous work in treating x as a continuous variable, and then relating this to future work on irrational numbers. This emphasis on the shift from a discrete relation to a continuous graph is emphasized throughout the teacher's notes in the chapter, for example: "Immediately a line is drawn, we are attaching meaning to the intermediate points and we must satisfy ourselves that each point on the line satisfies the relation" (p. 90).

The focus on the straight graph as representing a set of points all of which satisfy the relation is also evident in the pupil's textbook. So, the key move to the straight line graph is achieved by a consideration of the graphs over 3 pages (see Figure 1), first plotting only the first five whole number values, then plotting "more and more" (p.92) intermediate values such as (¾, 2¼). Then, after "join[ing] up the points by drawing a line with a ruler", the pupil is asked to consider whether the point (0, 0) lies on the line and "Is it true that for every point on the line, the second coordinate is always three times the first coordinate?" (p.93). Finally, the notation $y = 3x$ is introduced as the equation of the line and as an alternative and equivalent way of expressing the relation $x \rightarrow 3x$.

(Figure 1)

Misconceptions

Misconceptions as a term came to prominence in mathematics education in England during the late 1970s as a result of CSMS and other studies. Unsurprisingly, then, there is no explicit reference to misconceptions at any point in the teacher's guide. However, the book does highlight aspects of the mathematical ideas that may not be immediately clear to students, for example: "Some pupils do not realise that a line is a set of points" (p. 90). Moreover, in taking an explicitly mathematical approach and treating letters as numbers at all times, the *SMP* does appear to take the difficulties that students face in learning algebra seriously.

Multiple representations

A central aim is to "develop the idea of representing relations by means of diagrams" (p. 86). Whilst the pace of the SMP book might be considered a bit ponderous to a modern audience, this allows for representations and diagrams to be explored in some depth. A variety of them are used throughout the chapter: mapping diagrams, ordered pairs, arrow diagrams, tables of values and general (word) statements as well as straight line graphs. At several points the teacher's attention is drawn to the pedagogic features of specific representations, for example:

> Arrow diagrams on parallel number lines having the same scale emphasize the relation. ([This] shows the real nature of the relation, in this case, multiplication by 3.) This fact will be become more apparent to the pupils later in this chapter, when they are asked to find the relation between two sets of numbers. (p. 89)

Variation

The relation $x \rightarrow 3x$ is introduced in the context of a fairground wheel steadily turning, "deliberately chosen" as a relation between continuous variables. This example is contrasted for the teacher with "everyday examples involv[ing] discrete variables such as the relation between cost of postage and letter weight" (p. 89). Additionally, the examples used include relations with non-integer gradients (although gradient or steepness is not referred to), rational values of x, relations of the form $x + y = c$ (for numeric values of c) and two non-examples of linear relations for which the points "do not lie on a straight line…[and] can be joined up (with a smooth curve)" (p. 113): $x \rightarrow 24/x$ and $x \rightarrow x^2$. The rationale for these choices is discussed in the teacher's guide, for example, "until multiplication and division of directed numbers have been defined, we are not in a position to consider the graph of $y = ax$ in all four quadrants" (p. 105).

Purpose
There are few explicit references to the purposes for algebra. Although the relation is introduced through the "real-world" context of a fairground wheel, the context appears to be used primarily to provide a structure for the students to consider continuity rather than to represent or model this "real-world" context. Otherwise, algebra is treated largely as a topic from pure mathematics.

Framework Maths 8C
Framework Maths is a course aimed at ages 11–14 and covers the range of attainment. Differentiation is provided by three levels of textbooks—support, core and extension—which correspond to the dominant practice within English mathematics of grouping pupils in different classes by "ability" (Hodgen, 2007). *Book 8C* is the core book aimed at the middle range of attainment and covers a complete year's work.

The structure and timing of chapters follows exactly the suggested medium-term planning from the 2001 *Framework for Teaching Mathematics* (DfEE, 2001), with references to the learning objectives of the 2001 *Framework* on every page of the *Framework Maths Teacher's Guide*. In contrast to *SMP*'s prose style, each unit is structured consistently around a series of double-page lesson spreads. Additionally, the unit begins with an overview and ends with a summary, both focused on assessment. This focus on assessment of learning is also apparent in the corresponding homework book, in which national assessment questions are reproduced for each unit. On each double-page spread in the teacher's guide, the corresponding double-page spread in the pupil's book is reproduced, surrounded by a series of standard boxes providing guidance to the teacher. All the areas are of course important, but one consequence of this standardization is that the advice is at times rather cursory.

Mathematical coherence

Unit A3, "Functions and graphs", contains six lessons and is intended as just over a week's work. The first four lessons focus directly on moving from function machines and mapping diagrams to straight line graphs, whilst the later two lessons "extend to cover real-life graphs" (p. 85). In fact, the graphs in these two lessons are largely distance-time and temperature-time, and are rather more complex in that the lines or sets of points are not defined by a single relation. Hence, the extent to which these two lessons "extend" the more focused work of the previous four lessons is not clear. In particular, aside from the assertion, there is no discussion on this in the teacher's guide. In contrast to *SMP Book C*, it is only in the extension book, targeted at higher attaining pupils, that "smooth curves" such as $y = x^2$ are considered.

In comparison to *SMP Book C*, the unit in *Framework Maths* contains rather more advanced content. For example, whereas the former considers only "relations" of the form $y = ax$ and $y = x + a$ (where a is a numeric value and only for positive values of x), the latter refers to "functions" in all four quadrants.

Further, in *Framework Maths*, the general form $y = mx + c$ is explicitly considered. However, there is no discussion of any distinction between variables and parameters nor of any difficulties that these concepts might present for students. There is additionally no discussion that the line represents a set of points nor any hint, for either the teacher or the students, that the unit might be concerned with generalized number, or variable, rather than letters as specific unknowns. The parameters are treated somewhat unproblematically, although there is no definitive explicit definition given. For example, m is defined somewhat sloppily in the pupil's book as "a measure of the steepness or gradient" (p. 88) and in the teacher's guide as the "multiplier" (p. 98), with no additional commentary for the teacher. Similarly, c and $(0, c)$ are both defined as the y-intercept with no discussion that $(0, c)$ is a point that satisfies the relation. Indeed, the focus of the third lesson, straight line graphs, is on the graphs themselves as mathematical objects and on identifying families of graphs with either the same gradient or the same y-intercept, whilst in the associated homework students are given a grid containing six graphs (including one non-function, $x = -2$) and asked to match these to a set of corresponding equations (Capewell et al., 2003a, p. 55). There is only one small reference in the pupil's book (but not the teacher's guide) to the idea that the relation or function "describes all the values of x and y that lie on the line" (p. 88).

Hence, whilst the content is in some respects more advanced, it is at the same time less coherently presented and, unlike *SMP*, there is very limited consideration given to the key conceptual ideas to be taught.

Framework Maths has a strong procedural focus to the point that key conceptual ideas are treated purely in technical terms. So, for example, consideration is given to the number of points necessary to draw a straight line. The pupil's book states that "You only need to know two points to sketch a straight line" (p. 90), although rather confusingly, the advice in the teacher's guide is different as well as inconsistent: "Emphasise that you only need three points to be sure of your line" (p. 90) and "Emphasise that they need only two points with the third acting as a check" (p. 91). Further, it is suggested that students be advised to choose "small" and "easy" numbers (p. 90–91), presumably to avoid co-ordinate pairs that lie outside the bounds of pupils' co-ordinate grids, although this is not made explicit. The equivalent discussion in *SMP Book C* is more mathematical, more pedagogical, and anticipates potential difficulties that students may have:

> We are trying to get across the idea that it is not necessary to plot a large number of points in order to draw the graph of a linear relation. The minimum number of points needed is 'two' but 'three' is to be recommended as it provides a check: (i) on errors in calculation, (ii) on careless plotting of points. Also, for an accurate line, the three points should not be clustered close together. Many classes may not be ready for these ideas which can well be left until later. It is important, however, that pupils realise that whatever set of points satisfying a given relation they choose to plot, they will always get the same straight line. (p. 106)

We note that this final point relates the "technical" issue of the minimum number of points required back to the central conceptual idea of attaching meaning to all the co-ordinate pairs on the line. Given this emphasis on the minimum number of points coupled with the almost exclusive focus on integer values of x and y, the idea that the line represents a set of points all of which satisfy the relation, or function, is unlikely to be apparent to students, or indeed to some teachers.

Purpose

There is little attention devoted to purpose within this unit, although the unit commentary does address the purposes for real-life graphs, suggesting that teachers "discuss situations in which students have used graphs…[and] why graphs are such a popular form of presenting data" (p. 85). In earlier units, however, purposes for algebra are alluded to. For example, the book suggests that teachers should "emphasise that a broad range of people such as computer programmers might use algebra as it is quicker than writing out words in full" (p. 57). It seems unlikely that this assertion will address students' perceptions of the irrelevance of algebra, and there is no guidance of ways in which the students might come to appreciate the purpose and utility of algebra for themselves (Arcavi, 1994; Ainley, Pratt, & Hansen, 2006).

Misconceptions, multiple representations and variation

Somewhat surprisingly, we found little evidence of the influence of research in any of the remaining areas. Whilst there is a section headed "Misconceptions" on each double-page spread in the teacher's guide, the section refers to errors, or "silly mistakes" (p. 89), to be avoided rather than to conceptual difficulties that arise, for example, from over-generalizations. Similarly, there is no discussion on the use or choice of examples; indeed, for the mental starter, the teacher is largely left to generate examples with no guidance. Moreover, some examples are rather badly chosen. For example, in the second lesson, in which co-ordinate graphs are introduced, the first worked example (of two), $y = 2x - 2$ (p. 88), has a poor choice of numbers, which "confuses the role of variables" (Rowland, 2008, p. 155).

The book makes use of multiple representations, building from function machines, mapping diagrams, and tables of values to straight line graphs. However, by the third lesson, the emphasis is almost wholly on graphs, to the exclusion of other representations. Finally, we note that the pace of this topic, which is about twice as fast as that of *SMP Book C*, would allow only limited time for these issues.

Comparing the two textbooks

The comparison of the two textbooks suggests that *SMP Book C* is considerably more mathematically coherent than *Framework Maths*, in which there is limited explicit reference to the concept of variables or to the idea that the straight line is a set of points that all satisfy the relation. Moreover, in comparison, *Framework Maths* contains little evidence of the influence of research on the teaching and learning of algebra, whilst some of the key ideas from research such as misconceptions are misunderstood. This greater evidence of research in *SMP Book C* is perhaps surprising given that much of the research was carried out after the publication of the textbook series, although it may reflect the fact that many of the concerns of mathematics education research arose as a result of the development of textbooks such as *SMP* (Anderson, 1978). Thus, textbooks such as *SMP* may have been influential in setting the agendas for research.

Comparing textbooks in England to those of France and Germany, Haggarty and Pepin (2001) found that current English textbooks were routine and consisted of "mostly straightforward applications of the worked examples…[and] only rarely required deeper levels of thinking from pupils" (p. 172). Our analysis suggests that, compared to those of the 1970s, current English textbooks are likewise simple and routine with little requirement for deeper thinking, and this is compounded by the advice given in the teacher's guide. *Framework Maths* takes hardly any account at all of any of the substantial body of research on students' learning of algebra. Indeed, the approach is solidly focused on what Orton et al. (1995) call "the logic of mathematics", although even in this regard the focus is on a rather limited procedural logic in contrast to the richly mathematical approach in *SMP*.

Discussion

Our analysis suggests considerable differences between current English textbooks and those of 30 years ago. Of course, these differences might reflect different purposes and certainly current textbooks are directed at a great proportion of students taking and passing examinations at the end of compulsory schooling. In terms of examination performance, this aim has been successful, although as we have already noted, the increases in examination performance are not matched by increases in either understanding or participation. The SMP series was designed to be educative (Davis & Krajcik, 2005). The aim was to provide a resource that could both be used independently by students on their own and also support mathematics teachers in better understanding school mathematics (Thwaites, 1972). We have no evidence as to the efficacy of these aims, although, at least as regards teachers, our analysis does suggest that the SMP series does have more educative potential than the current textbook. A key feature was a coherent presentation of the mathematics, centred upon two key ideas: the move to continuous variables, and the graph as representing a series of points. In contrast, *Framework Maths*, following the Mathematics Framework (DfEE, 2001), contains a number of much more fragmentary and only loosely connected learning objectives.

The case of mathematics in England has generated a great deal of interest internationally because of the apparent "success" of national policy and recent rises in TIMSS performance (Mullis et al., 2008). We are not arguing that school mathematics in England during the 1970s was better—it probably was not. However, we do suggest some caution in such international comparisons. National policy has had some unintended consequences. It is unlikely, for example, that there was any intention to reduce the time and in-school trialling devoted to the production of textbooks, to reduce the influence of academics in mathematics education with knowledge of research, or to increase publishers' focus on marketing. Yet, this is exactly what has happened.

There is something of a wake-up call for the mathematics education research community here. That the current textbook contains so little evidence of research suggests a considerable dissemination problem. The international literature suggests that, whilst England may be a special case, the problem is replicated in many other countries (Stein, Remillard, & Smith, 2007). There are, of course, many instances of research-informed curriculum materials. Swan's (2005) excellent materials, for example, have been distributed to all secondary schools in England, but their use is patchy and appears to be dependent on the influence and enthusiasm of a local advisory teacher. We suggest that there is an urgent need to investigate not simply how to design research-informed textbooks, but more importantly how such materials can appeal to teachers and schools and to make use of both practical and theoretical approaches to modern design that are educative.

There is an increasing body of research and expertise in the design and development of teaching materials and approaches (e.g., Ruthven, Laborde, Leach, & Tiberghien, 2009). This body of work demonstrates the importance of an iterative process of classroom trials in evaluating, adapting and improving materials. Indeed, it is commonplace to refer to this process as one of *didactical engineering* (Wittman, 1995). However, without the input of experts in mathematics education research, even well-trialled textbooks are almost doomed to fail. Clearly it is important that policy makers, publishers and others take mathematics education researchers seriously. However, this also places a duty on the mathematics education research community. We cannot simply blame others for ignoring our research. The community itself needs to value the "re-working" of research findings into textbooks as importantly as the original research.

Notes

1. This figure aggregates teachers' self-reports of the use of textbooks as a primary basis for lessons (60%) and as a supplementary resource (34%) for teaching mathematics (Mullis et al., 2008, p. 291). Only 6% of teachers internationally report that they do not use textbooks.
2. Compulsory schooling in England is divided into four Key Stages: KS1 and KS2 in primary (ages 5–7 and 7–11, respectively), and KS3 and KS4 in secondary (ages 11–14 and 14–16, respectively). KS3 consists of three year groups: Year 7, Year 8 and Year 9. Year 8 covers ages 12–13 and is roughly equivalent to Grade 7 in most other national systems.
3. General Certificate of Secondary Education (GCSE) examinations are taken at the end of compulsory education (age 16) in England.
4. These early findings should be interpreted with some caution as they may differ somewhat from the final outcomes, especially because they are based only on the 2008 sample, which was a little higher attaining than the national average. It is likely that, when the full sample is analysed, current levels of understanding in algebra will be slightly lower. The increase at Year 8 is likely to

be due to algebra appearing earlier in the curriculum than in the 1970s and to the slightly above average sample in 2008.

5. In practice, many of the higher attaining students were taught using the previous numbered textbook series that had been designed for selective schools.

6. Prior to 1987, England had a two-tier examination system at the end of formal schooling. The GCE O-level (General Certificate of Education, Ordinary Level) was aimed at the highest attaining students, with CSE (Certificate of Secondary Education) qualification aimed at the remainder. Grade C of the GCSE was set at equivalent to O-level Grade C and CSE Grade 1.

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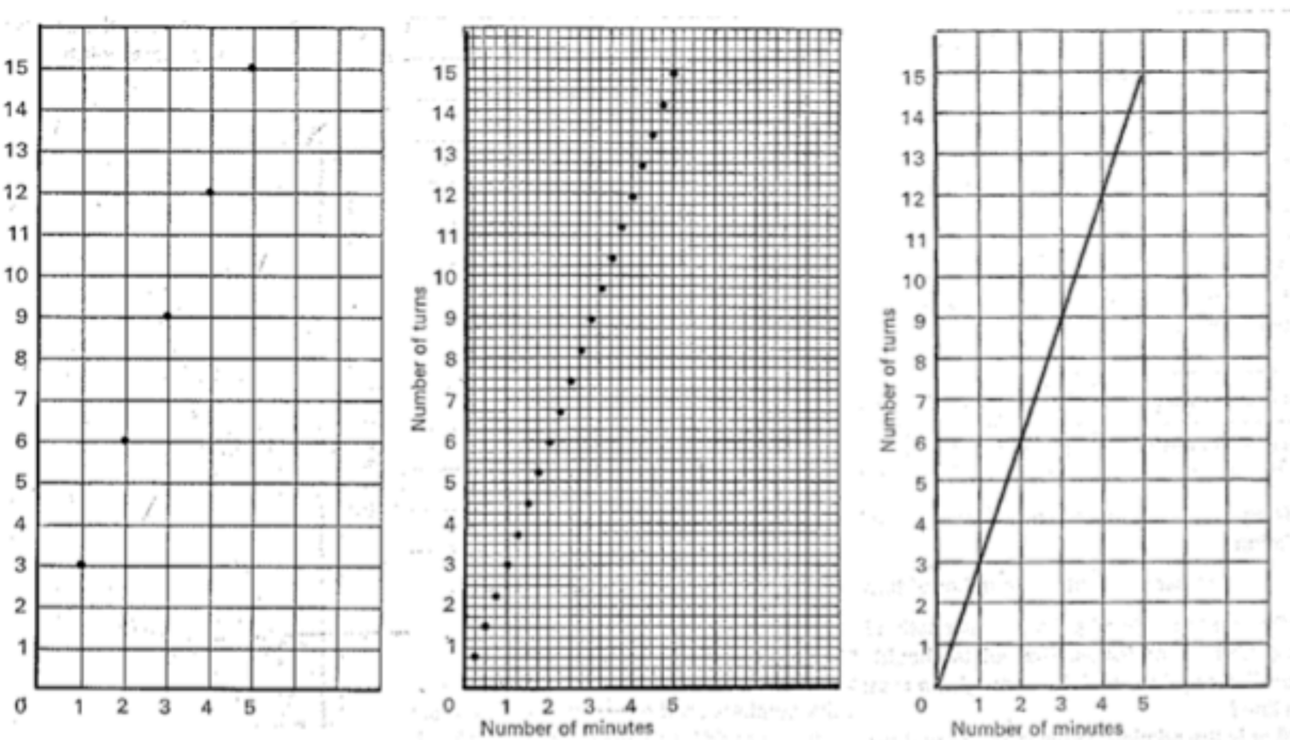

Figure 1. Figures 3, 4 and 5 from *SMP Book C* (pp. 91–93).