Attainment, participation and the need for post-16 mathematics education: Findings from the ICCAMS Project

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In this briefing paper, I show that there is an urgent need to increase attainment and participation in mathematics. In particular, there are currently few options available for students aside from the traditional A-level. I suggest how additional post-16 mathematics qualifications could address this problem.

Attainment in algebra and multiplicative reasoning needs to improve

Increasing Competence and Confidence in Algebra and Multiplicative Structures (ICCAMS) is a 4-year research project funded by the Economic and Social Research Council as part of their Targeted Initiative on Science and Mathematics Education (TISME). ICCAMS set out to investigate attainment and engagement in two areas of mathematics at Key Stage 3, algebra and ratio. These areas of mathematics are key foundations for further study in mathematics, science and other STEM areas as well as other numerate disciplines such as economics, whilst multiplicative reasoning is additionally essential for many aspects of modern life (such as understanding risk). Both are also areas of national concern among STEM academics.

Phase 1 of the ICCAMS project consisted of a survey of 11-14 years olds' understandings of algebra, ratio and decimals, and their attitudes to mathematics using a stratified random sample of schools In England. ICCAMS used tests first administered by the Concepts in Secondary Mathematics and Science (CSMS) study in the 1970s allowing a comparison of standards over time[1]. Here, for reasons of space, I focus largely on the results for the ratio test at Year 9 (age 14).

In the 1970s analysis, items were selected empirically from each test to form a series of hierarchical levels of difficulty. Broadly, students achieving Level 1 can successfully solve problems involving doubling, trebling and halving. However, when presented with scaling or conversion problems involving non-whole number ratios, they tend to see these as additive not multiplicative. For example, if asked to convert currency in the ratio 4:6, they would tend to see this as an "add 2" relationship rather than "multiply by 1.5". In contrast, by Level 4, students can successfully solve enlargement problems, although they may still lack some numerical fluency.

The results of the comparison are shown in Figure 1[2]. These indicate a slight decline in attainment over the 30 year period. There is marked increase in the proportion of students who do not achieve Level 1 (from 7% in 1976 to 15% in 2008/9) and of those achieving Level 1 or below (from 56% in 1976 to 65% in 2008/9). There is also a statistically significant decrease in the proportion of the highest attainers at Level 4 (from 9% in 1976 to 6% in 2008/9). This decline amongst the highest attainers is all the more significant when considered in the context of A-level participation numbers: more than half the 13% who will go on to take AS mathematics at 16 appear to have significant difficulties with ratio at age 14.

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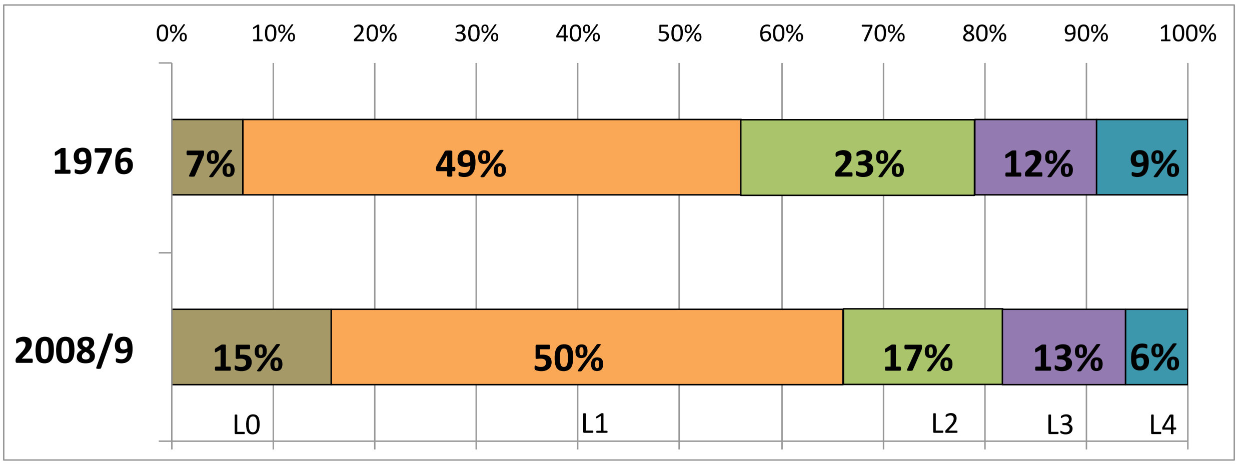

Figure 1: A comparison of the distribution of Year 9 students at different levels of performance in the ratio test in 1976 (N=1595) and in 2008/9 (N=767). L0 = Level 0; L1 = Level 1 etc.

The results for algebra and decimals were similar to those for ratio except that there was a little more progress in the middle of the attainment range for decimals.

The ICCAMS results suggest that there is little evidence for the sort of step-change in mathematical attainment that might be suggested by the apparent improvements in GCSE or Key Stage 2 results, and indeed the general trend is for results to be somewhat lower than in the 1970s. We discuss the issue of standards over time in several academic papers[3]. However, the more substantive concern is that the ICCAMS results suggest that the vast majority of students, including some who will take advanced mathematics post-16, have very significant difficulties in the key mathematical areas of algebra and multiplicative reasoning at age 14. It is unlikely that these difficulties can be resolved in the intervening two-year period.

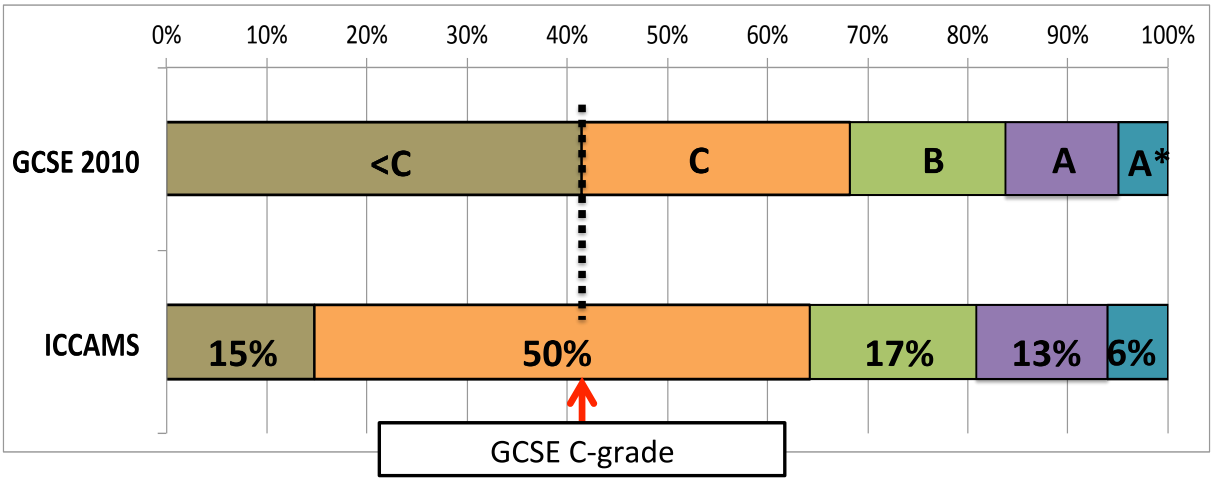

Figure 2: The distribution of attainment for Year 9 students in the ICCAMS Ratio test for 2008/9 compared to the distribution of GCSE results in 2010.

In Figure 2, the ICCAMS results are compared to the distribution of GCSE results in 2010. Some caution needs to be exercised in interpreting this comparison. Nevertheless, the comparison gives an indication of how the indication of how the distribution of attainment at age 14 relates to GCSE grades at 16. It is a very significant concern that many of the students, who go on to achieve a C grade at GCSE, are struggling with even the most basic notion of ratio at age 14. In short, the job of mathematics education is not complete for most students by the age of 16. Whilst there is certainly a need to improve attainment pre-16, it is vital that more students are encouraged to continue to study mathematics post-16. Currently, in England, however, there are few options for those who either are no able or do not want to study A-level mathematics.

Participation in post-16 mathematics in England is unusually low internationally

In a separate study funded by the Nuffield Foundation, "Is the UK an outlier? An international comparison of upper secondary participation in mathematics education", we examined post-16 participation in the UK nations in comparison to a sample of 20 other educational systems internationally[4]. A selection of these results is shown in Table 1.

	Participation in …	
	Some Mathematics	Advanced Mathematics
Japan	All	85%
Korea	All	57%
Finland	All	20%
Russia	All	1%
Germany	Most	11%
New Zealand	Many	41%
Singapore	Many	31%
Scotland	Some	23%
England	Few	13%

Table 1: A comparison of participation in basic and advanced mathematics in selected systems internationally. Advanced Mathematics is equivalent to at least AS-level pure mathematics in England. All = 95-100%; Most = 80-95%; Many = 51-79%; Some = 20-50%; Few < 20%.

These results indicate that participation in any mathematics in England post-16 is unusually low in comparison to other systems. Indeed, in the vast majority of other systems more than 50% of students continue to study at least some mathematics, whereas in England, the proportion is less than 20%. For advanced mathematics, participation is also low.

A key factor in England's low participation is a lack of provision. For those students who achieve a C grade at GCSE, there is almost no option to study mathematics. Indeed, most schools require students to achieve at least an A grade at GCSE to study mathematics. For those students who do not achieve a C grade, the provision is patchy and often remedial. As a result, only an additional 3% of the cohort achieve a C grade equivalent by age 19[5]. (See Figure 3.)

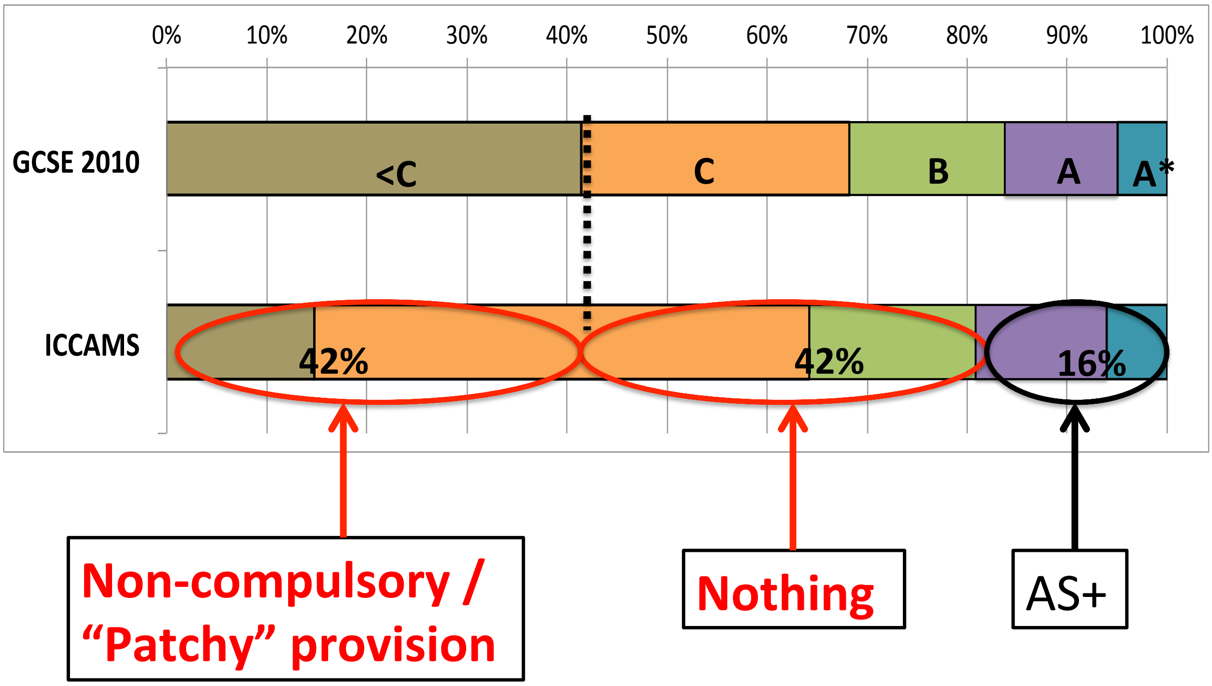

Figure 3: The post-16 options in mathematics for students at different levels of attainment for Year 9 students in the ICCAMS Ratio test for 2008/9 compared to the distribution of GCSE results in 2010.

Both New Zealand and Scotland offer valuable lessons for England in terms of advanced mathematics participation. Both are systems in which, like England, students are free to choose (or not to choose mathematics) in upper secondary education, but both have higher participation rates than England. Scotland have achieved this by encouraging students to take more subjects in upper secondary (5-6 subjects rather than the 3-4 which is the norm in England) whilst also reducing the range of subjects from which students can choose[6]. In addition, early entry to Standard Grade (the Scottish equivalent to GCSE) has been discouraged allowing students more opportunity to develop a fuller understanding of mathematics.

New Zealand, however, outperforms both England and Scotland in terms of participation in advanced mathematics (41% in comparison to England’s 13% and Scotland’s 23%). In New Zealand, students can choose from two advanced mathematics options: a traditional mathematics with calculus option similar to A-level in England (and Higher in Scotland), and a mathematics and statistics option. 27% of the cohort choose the mathematics and statistics option, whilst the proportion studying the traditional course is slightly higher than England at 15%. (See Figure 4.) A further factor may be New Zealand’s higher performance in the OECD’s PISA survey: 41% of 15 year olds in New Zealand achieve Level 4 or higher in PISA in comparison to 33% in England[7]. However, whilst the figures indicate that New Zealand attracts most of these relatively high attaining students to study advanced mathematics in upper secondary, England attracts less than half of this high attaining group.

Improving attainment is a factor in increasing participation, but it is also important to provide a wider range of options for advanced mathematics study. Indeed, given the entry requirements for A-level mathematics in most schools and colleges, England may well be approaching the upper limit for participation in a traditional mathematics with calculus A-level[8]. Hence, at least in the medium term any further increases in participation are likely to require an advanced mathematics option that is both more appropriate and more engaging to these students.

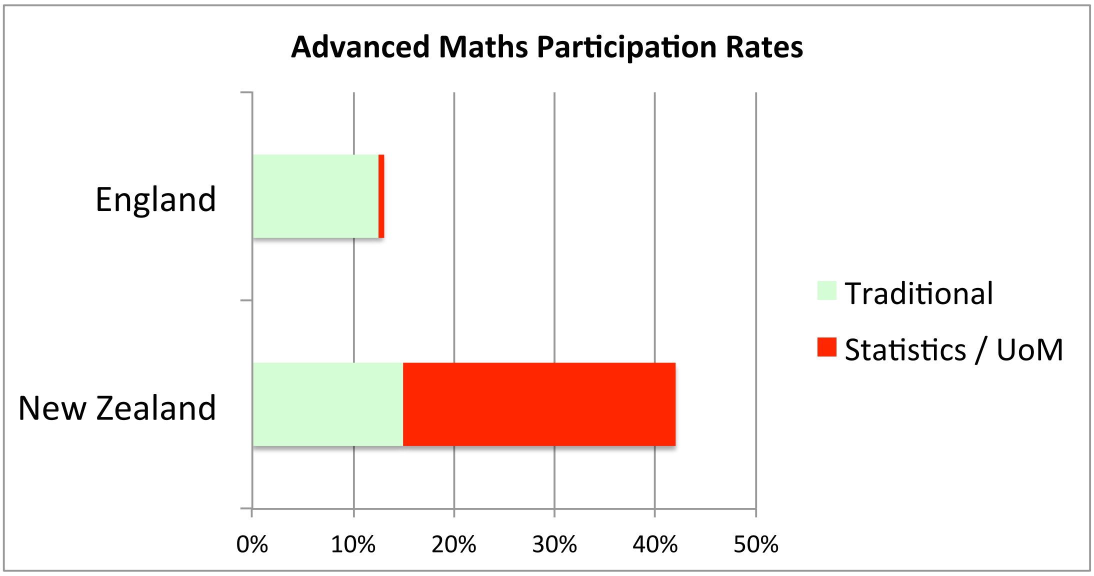

Figure 4: A comparison of participation in advanced mathematics in England and New Zealand. Both systems offer a traditional mathematics with calculus qualification (indicated in blue). In New Zealand, a substantial proportion of student take an alternative advanced qualification in mathematics and statistics (indicated in red). The closest equivalent to this in England are the Free-Standing Mathematics Qualifications (FSMQs) / Use of Mathematics AS-level.

Improving post-16 mathematics provision

A greater range of options is required in post-16 mathematics. The analysis from the international comparative study suggests the following in addition to the traditional A-level:

- Introduce an advanced mathematics and statistics route as in New Zealand. This could be aimed aimed at students who are intending to study numerate disciplines such as economics, geography and biology. The closest existing model to this in England are Free Standing Mathematics Qualifications / AS Use of Maths, although these are not widely available. Research indicates that lower attaining students taking this option are less likely to drop out of mathematics[9].
- Enable the existing AS mathematics course to be studied over 2 years. This would be attractive to some lower attaining students by allowing them more time to engage with advanced mathematics. Current funding arrangements discourage this.
- Introduce a "basic" mathematics course to provide some mathematics particularly aimed at students who have achieved a C grade at GCSE and aimed particularly at the application of mathematics in topics related to students' other courses. This course must be examined in order to be taken seriously by schools, colleges and students.
- Make GCSE mathematics or equivalent compulsory for those who have not achieved a C grade at GCSE.

In addition, numerate disciplines need to require (and examine) more mathematics from students in terms of the range and difficulty of concepts[10], whilst universities (and employers) need to indicate to students the value of mathematics.

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